Recent Advances in Surface Wave Methods for Geotechnical Site Characterization

Récents Développements des Méthodes d'Ondes de Surface pour la Caractérisation des Sites Géotechniques

GLENN J. RIX, Georgia Institute of Technology
CARLO G. LAI, Studio Geotecnico Italiano
M. CATALINA OROZCO, Georgia Institute of Technology
GREGORY L. HEBELER, Georgia Institute of Technology
VITANTONIO ROMA, Politecnico di Torino

ABSTRACT: Surface wave methods are non-invasive seismic techniques to determine the shear wave velocity and shear damping ratio profiles at sites. Recent developments including the use of multiple-receiver arrays, active and passive sources, simple methods to account for multiple modes of propagation, robust inversion algorithms, Rayleigh wave attenuation and damping measurements, and simultaneous measurement and inversion of dispersion and attenuation curves have greatly enhanced the capabilities of surface wave methods. In this article these improvements are briefly described and illustrated via examples.

RÉSUMÉ: Les méthodes basées sur l'emploie d'ondes de surfaces sont tecniques sismiques non desctructives qui sont utilisées pour déterminer les profils de vitesse d'ondes de cisaillement e du coefficient d'amortissement in situ. Récents développements comme l'emploie d'alignement des récepteurs multiples, sources actives e passives, méthodes simplifiées pour tenir compte de la propagation multimodale, solides algorithmes d'inversion, mesures d'atténuation e d'amortissement d'ondes de Rayleigh, mésure et inversion simultanée des curves de dispersion e d'atténuation, ont élargi enormement les potentialités des méthodes basées sur l'emploie d'ondes de surfaces. Dans cet article ces développements sont brièvement décrits et illustrés avec des examples.

1 INTRODUCTION

In situ seismic measurements based on surface wave propagation have been used in geotechnical engineering for the past 50 years. Early versions of surface wave tests utilized steady-state sources, simple methods of calculating surface wave velocity, and empirical inversion algorithms to estimate the shear wave velocity profile. In the 1970s, the advent of portable dynamic signal analyzers capable of performing Fourier analysis enabled the use of transient sources that reduced testing times considerably. Concurrently, the increasing availability of computers made the use of theoretically based inversion algorithms routinely possible. The result of these advances was the Spectral-Analysis-of-Surface-Waves (SASW) method (Stokoe et al. 1994).

Recently, several new developments have expanded the capabilities of surface wave methods for site characterization. These developments include:

1. the use of spatial array processing techniques for measuring Rayleigh phase wave velocity;
2. the use of low-frequency Rayleigh waves from passive sources such as microtremors and/or cultural activities to complement active-source tests;
3. a simple, closed-form method for calculating the effective phase velocity of multi-mode Rayleigh waves;
4. the use of smoothing constraints to improve the robustness of inversions;
5. a method to determine the shear damping ratio profile from observations of Rayleigh wave spatial attenuation; and
6. simultaneous measurement and inversion of Rayleigh wave phase velocity and attenuation using a viscoelastic constitutive model of small-strain soil behavior.

In this paper we review these developments to summarize the state-of-the-art in engineering surface wave test methods. Although there have been many innovative new applications of surface wave methods, our scope is limited to developments related to test procedures and interpretation.

2 SPATIAL ARRAY PROCESSING TECHNIQUES

2.1 *Frequency-wavenumber methods*

Several investigators have employed frequency-wavenumber (*f-k*) methods to yield robust estimates of surface wave dispersion curves (Gabriels et al. 1987, Tokimatsu 1995, Zywicki 1999, Park et al. 1999, Foti 2000, Liu et al. 2000). These methods typically utilize multiple receivers arranged in a one-dimensional (i.e., linear) array for active measurements or a two-dimensional array for passive measurements. The *f-k* spectrum is estimated via a process called *beamforming*. Let $y_m(t)$ be the time history observed at the m^{th} receiver $(m = 1,\ldots,M)$, which is located at position $\mathbf{x}_m = (x_m, y_m)$. $\mathbf{Y}(\omega) = \mathrm{col}[Y_1(\omega),\ldots,Y_M(\omega)]$ denotes the column vector containing the Fourier Transforms of the signals observed at each receiver. If desired, the signals may be weighted via a diagonal matrix $\mathbf{W}(\omega) = \mathrm{diag}[w_1(\omega),\ldots,w_M(\omega)]$. Also define a *steering vector*:

$$\mathbf{e}(\mathbf{k}) = \mathrm{col}[\exp(-i\mathbf{k}\cdot\mathbf{x}_1),\ldots,\exp(-i\mathbf{k}\cdot\mathbf{x}_M)] \tag{1}$$

where $\mathbf{k} = (k_x, k_y)$ is the vector wavenumber. The *steered response power* (Johnson & Dudgeon 1993) is given by:

$$P(\mathbf{k},\omega) = \mathbf{e}^H\mathbf{W}\mathbf{Y}\mathbf{Y}^H\mathbf{W}^H\mathbf{e} = \mathbf{e}^H\mathbf{W}\mathbf{R}\mathbf{W}^H\mathbf{e} \tag{2}$$

where H denotes the Hermitian transpose of the vector and $\mathbf{R}$ is the spatio-spectral correlation matrix:

$$\mathbf{R}(\omega) = \begin{bmatrix} \hat{G}_{11}(\omega) & \hat{G}_{12}(\omega) & \cdots & \hat{G}_{1M}(\omega) \\ \hat{G}_{21}(\omega) & \hat{G}_{22}(\omega) & \cdots & \hat{G}_{2M}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{G}_{M1}(\omega) & \hat{G}_{M2}(\omega) & \cdots & \hat{G}_{MM}(\omega) \end{bmatrix} \tag{3}$$

Each term in the spatio-spectral matrix is the cross-power spectrum between two receivers.

At a given frequency ω, peaks in the steered power response correspond to Rayleigh wave modes propagating across the array of receivers. The phase velocity of these surface waves is given by:

$$V_R = \frac{\omega}{|\mathbf{k}|} \tag{4}$$

Equation 2 is the basis of a variety of *f-k* methods including the conventional frequency-domain beamformer and several adaptive approaches such as the minimum variance method (Capon 1969) and signal-noise subspace methods.

Figure 1 compares the dispersion curve calculated using the *f-k* method with the composite dispersion curve from the conventional, two-station method used in SASW testing. The *f-k* dispersion curve was determined using an active harmonic source and a linear array of $M = 15$ receivers. Note that the *f-k* dispersion curve is more regular and, thus, corresponds more closely with the expected behavior of Rayleigh waves in vertically heterogeneous media. In practical terms, *f-k* dispersion curves are more easily inverted to obtain the shear wave velocity profile (Hebeler 2000).

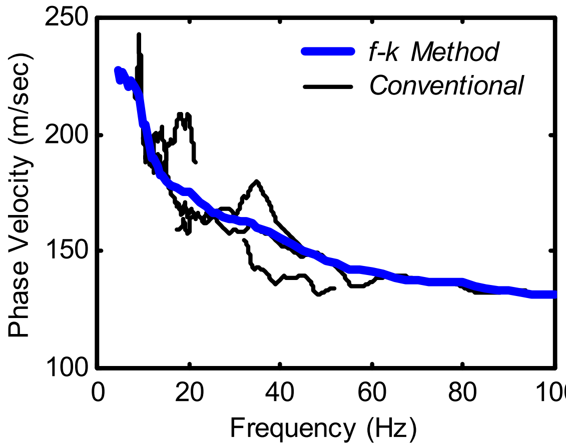

Figure 1. Comparison of *f-k* dispersion curve with composite dispersion curve from the two-station method.

2.2 *Combined active-passive measurements*

Using multi-receiver *f-k* methods, it is possible to combine active and passive surface wave measurements to yield a dispersion curve that spans a broader range of frequencies. Active measurements are performed using a linear array with the source (i.e., an electro-mechanical shaker) positioned at one end of the array. The frequency range for active measurements is approximately 4 to 100 Hz depending on the characteristics of the source and site conditions. Passive measurements utilize the ambient wavefield generated by microtremors and/or cultural activities. A two-dimensional array is used since the location of the source(s) is often unknown. It is assumed that vertical particle motions measured at the receivers are due to plane Rayleigh waves propagating across the receiver array. Since the frequencies of the passive Rayleigh waves are typically lower than the active waves, the addition of the passive measurements allows profiling to much greater depths. Figure 2 shows the combined dispersion curve; adding the passive dispersion measurements increased the maximum resolvable wavelength from approximately 60 m to 250 m and enabled a much deeper shear wave velocity profile to be obtained at this site (Hebeler 2000).

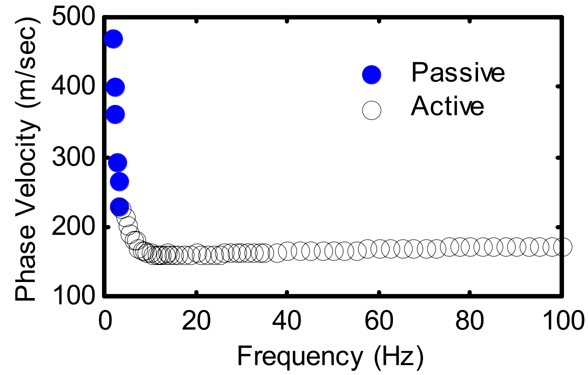

Figure 2. Combined active-passive dispersion curve.

3 MULTI-MODE EFFECTIVE PHASE VELOCITY

A common practice in engineering surface wave testing is to use a theoretical dispersion curve including only the fundamental mode of propagation in the inversion process. This method of analysis is often called a 2-D analysis of surface waves (Roësset et al., 1991). Since the experimental dispersion curve contains, in general, the contribution of several modes of propagation, this approach is inconsistent. The results provided by a 2-D analysis are generally satisfactory for normally dispersive (i.e. regular) shear wave velocity profiles. However, for inversely dispersive (i.e., irregular) profiles the contribution of higher propagation modes may be important and usually cannot be neglected (Gucunski & Woods 1992, Tokimatsu 1995).

A 3-D surface wave analysis (Roësset et al. 1991, Ganji et al. 1998) accounts for multi-mode wave propagation and overcomes this limitation. It is based on reproducing the actual surface wave test by means of a numerical simulation. Hence the theoretical phase velocities are computed from phase differences between theoretical displacements calculated at the same receiver locations used in the experiment. The 3-D method is exact, but it requires the solution of a boundary value problem of 3-D elastodynamics. This can be computationally expensive, particularly if one wants to include the contributions of the body wave field. Another drawback of a 3-D analysis is that the partial derivatives required for solving the non-linear inverse problem are computed numerically. Computation of numerical partial derivatives is a notoriously ill-conditioned problem and is also computationally expensive.

Lai & Rix (1999) developed a new approach to multi-mode surface wave propagation that combines the simplicity of a 2-D analysis with the robustness of a 3-D analysis. From the property that the equation of a wave front represents the locus of points having constant phase, they derived an explicit, analytical expression for the effective Rayleigh phase velocity. The effective Rayleigh phase velocity may be entirely determined from the solution of the homogeneous Rayleigh eigenproblem, and, thus, requires no additional effort compared to a 2-D analysis. At the same time, use of the effective Rayleigh phase velocity achieves many of the advantages of a 3-D analysis because it includes contributions from multiple surface wave modes. Moreover, partial derivatives of the effective Rayleigh phase velocity with respect to the shear wave velocity of the medium are efficiently and accurately calculated using closed-form analytical expressions. These partial derivatives are used for inversions based on the effective Rayleigh phase velocity.

Figure 3 compares the modal and effective Rayleigh phase velocity curves for an inversely dispersive profile (Tokimatsu et al. 1992, Case 2). At frequencies less than about 30 Hz the effective velocity closely matches the fundamental mode, but at higher frequencies the effective phase velocity reflects the contribution of higher modes.

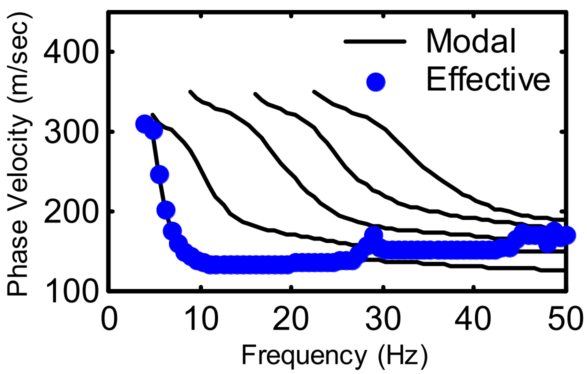

Figure 3. Comparison of modal and effective Rayleigh phase velocities.

4 INVERSION

Rayleigh inverse problems, like many other non-linear inverse problems, are ill-posed which implies that a given experimental dispersion curve may correspond to more than one shear wave velocity profile. From a mathematical point of view, non-uniqueness in the solution of an inverse problem is either caused by a lack of information to constrain its solution or, alternatively, because the available information content is not independent (Lai 1998). In light of this, adding information is often a good strategy to enforce uniqueness in the solution of an inverse problem. One common strategy is to constrain the solution to have certain global features such as smoothness and regularity. An example of such a strategy is the constrained, non-linear least squares algorithm proposed by Constable et al. (1987). The algorithm seeks to find the smoothest profile (in the sense of the first Fréchet derivative) of shear wave velocities subject to the constraint of a specified misfit between the experimental and theoretical dispersion curves. Their approach is motivated by the observation that inversions performed with classic least squares techniques often lead to physically unreasonable profiles of model parameters. The algorithm requires a definition of smoothness or its converse roughness of a candidate solution (in our case the V_s profile). In a layered soil profile, roughness may be defined by the following expression:

$$R_1 = (\partial V_s)^H (\partial V_s) \tag{5}$$

where V_S is an *nl x 1* vector of shear wave velocities of the layers (*nl* is the number of layers) and ∂ is an *nl x nl* matrix defining the two-point-central finite difference operator (Constable et al., 1987).

The error misfit ε^2 between the measured and the predicted Rayleigh phase velocities may be written as follows:

$$\varepsilon^2 = \left[\mathbf{W}V_R^{exp} - \mathbf{W}V_R^{theo}\right]^H \cdot \left[\mathbf{W}V_R^{exp} - \mathbf{W}V_R^{theo}\right] \tag{6}$$

where $V_R^{\ exp}$ is an *nf x 1* vector of experimental Rayleigh phase velocities (*nf* is the number of frequencies), $V_R^{\ theo}$ is an *nf x 1* vector of theoretical modal or effective Rayleigh phase velocities and **W** is a diagonal *nl x nl* matrix:

$$\mathbf{W} = \mathrm{diag}\{1/\sigma_1, 1/\sigma_2, \ldots\ldots, 1/\sigma_{nl}\} \tag{7}$$

containing the uncertainties associated with the experimental data. The solution of the non-linear Rayleigh inverse problem consists of finding a vector V_s that minimizes R_1 with the constraint that the residual error ε^2 be equal to a specified value $\varepsilon^{2(*)}$ that is acceptable in light of the uncertainties. The method of Lagrange multipliers is employed to solve this constrained optimization problem resulting in:

$$V_s = \left[\mu\partial^T\partial + \left(\mathbf{W}J_{V_{s0}}\right)^H \mathbf{W}J_{V_{s0}}\right]^{-1} \cdot \left(\mathbf{W}J_{V_{s0}}\right)^H \mathbf{W}\left[J_{V_{s0}} \cdot V_{s0} + (V_R - V_{R0})\right] \tag{8}$$

where μ is the Lagrange multiplier, which may be interpreted as a smoothing parameter, and V_{R0} are the modal or effective Rayleigh phase velocities obtained from the solution of the Rayleigh forward problem with $V_s = V_{s0}$. The term J_{Vso} is the *nf x nl* Jacobian matrix whose elements are the partial derivatives of the modal or effective Rayleigh phase velocities with respect to the shear wave velocities of the layers. These partial derivatives may be computed with closed-form, analytical expressions by using the variational principle of Rayleigh waves, thereby making the calculation of the Jacobian J_{Vso} much more efficient and accurate (Lai 1998).

Equation 8 is used iteratively to refine the estimated shear wave velocity profile V_{s0} until convergence. The smoothing parameter μ must be chosen so that the specified residual error $\varepsilon^{2(*)}$ is obtained. Figure 4 compares the theoretical dispersion curve corresponding to the final shear wave velocity profile with the active-passive experimental dispersion data presented previously. The excellent match is indicative of a successful inversion.

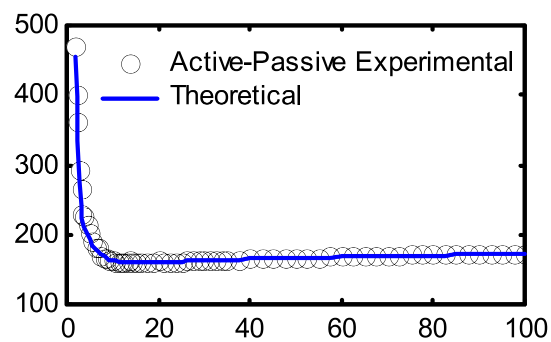

Figure 4. Comparison of experimental and theoretical dispersion curves

5 ATTENUATION AND DAMPING MEASUREMENTS

Surface wave methods may also be used to determine the material damping ratio of soils in situ by measuring the spatial attenuation of Rayleigh wave amplitudes (Jongmans & Demanet 1993, Rix et al. 2000). The first step in such an analysis is to determine an experimental attenuation curve showing the variation of the Rayleigh attenuation coefficient α_R with frequency. The Rayleigh attenuation curve is usually calculated using the amplitude of the particle motion recorded at various offsets from a transient or preferably harmonic source. The receivers must have identical frequency response characteristics. The accuracy of the measured attenuation coefficients is improved by properly accounting for the geometric attenuation of multi-mode Rayleigh waves.

Once the frequency-dependent attenuation coefficients are determined, the shear damping ratio profile is calculated by means of a constrained linear inversion analysis using the following expression:

$$\alpha_R(\omega) = \frac{\omega}{V_R^2}\left\{\sum_{j=1}^{nl}\left[V_{p_j}\left(\frac{\partial V_R}{\partial V_p}\right)_j K + V_{s_j}\left(\frac{\partial V_R}{\partial V_s}\right)_j\right] D_{s_j}\right\} \tag{9}$$

where V_{pj} and D_{sj} are respectively the compression phase velocity and the shear damping ratio of layer *j*, and *K* is the ratio of the compression to the shear damping ratio. Rix et al. (2000) demonstrated that the results of surface wave damping measurements agreed well with independent laboratory and in situ test results at the Treasure Island National Geotechnical Experimentation Site.

6 SIMULTANEOUS MEASUREMENT AND INVERSION OF DISPERSION AND ATTENUATION CURVES

Until recently, the two problems of determining the shear wave velocity and shear damping ratio profiles at a site have been considered separately and therefore have been uncoupled. Rix & Lai (1998) developed a method to simultaneously invert the experimental Rayleigh dispersion and attenuation curves to obtain the shear wave velocity and shear damping ratio profiles at a site. The new approach is motivated by the recognition that, in dissipative media, Rayleigh phase velocity and attenuation are not independent as a result of material dispersion. Therefore, a coupled analysis of dispersion and attenuation is a more robust, fundamentally correct approach.

A simple constitutive model that accounts for the most important dissipation processes occurring at small deformations in soils is linear viscoelasticity. An attractive feature of this theory is the elastic-viscoelastic correspondence principle that allows the solution of an elastic boundary value problem to be easily converted into the solution of the viscoelastic boundary value problem by using complex-valued seismic wave velocities:

$$V_{\kappa}{}^{*} = V_{\kappa}\sqrt{\frac{1+\sqrt{1+4D_{\kappa}^{2}}}{2\left(1+4D_{\kappa}^{2}\right)}}\sqrt{1+2iD_{\kappa}} \tag{10}$$

where $\kappa = S,P,R$ denotes the shear, compression, and Rayleigh wave phase velocities, respectively. In the viscoelastic case the Rayleigh dispersion equation is complex-valued, and therefore a different technique is needed to find its complex-valued roots. Rix & Lai (1998) employed an algorithm that is based on Cauchy's residue theorem from complex analysis.

Simultaneous inversion of surface wave velocity and attenuation data is superior to the traditional uncoupled analysis in the sense that it considers the effect produced by material dispersion and, secondly, improves the well posedness of the inverse problem. When combined with a procedure to simultaneously measure the dispersion and attenuation curves (Rix et al., 2001), the coupled approach forms a robust, consistent approach for measuring both shear wave velocity and shear damping ratio profiles.

7 CONCLUSIONS

Surface wave methods continue to evolve and become more capable. Initial efforts to use surface waves relied on simple test procedures and interpretive methods. The SASW method was an important advance in engineering surface wave methods because of the use of real-time Fourier analysis and theoretically based inversion algorithms. In recent years, surface wave methods have continued to develop through the use of powerful and robust signal processing, wave propagation and inversion techniques. As a result of these developments, geotechnical engineers have a powerful, flexible technique for site characterization.

8 ACKNOWLEDGEMENTS

The National Science Foundation supported the work presented herein under Award Nos. CMS-9402358 and CMS-9616013. The third author is supported by a National Science Foundation Graduate Fellowship. The authors would like to express their appreciation to A. Galigani and S. Pedretti for the French translation of the abstract.

9 REFERENCES

Capon, J. 1969. High-resolution frequency-wavenumber spectrum analysis. *Proceedings of the IEEE* 57(8): 1408-1418.

Constable, S.C., Parker, R. L., & Constable, C.G., 1987. Occam's inversion: a practical algorithm for generating smooth models from electromagnetic sounding data. *Geophysics* 52: 289 - 300.

Foti, S. 2000. Multistation methods for geotechnical characterization using surface waves. Ph.D. dissertation. Politecnico di Torino.

Gabriels, P., Snieder, R. & Nolet, G. 1987. In-situ measurements of shear-wave velocity in sediments with higher-mode Rayleigh waves. *Geophysical Prospecting* 35(2): 187-196.

Ganji, V., Gucunski, N., & Nazarian, S., 1998. Automated inversion procedure for spectral analysis of surface waves. *Journal of Geotechnical and Geoenvironmental Engineering* 124(8): 757-770.

Gucunski, N. & Woods, R.D., 1992. Numerical simulation of the SASW test. *Soil Dynamics and Earthquake Engineering* 11: 213-227.

Hebeler, G.L. 2000. Site characterization in Shelby County, Tennessee using advanced surface wave methods. Master's thesis. Georgia Institute of Technology.

Johnson, D.H. & Dudgeon, D.E. 1993. *Array signal processing: concepts and techniques*. Englewood Cliffs: Prentice-Hall.

Jongmans, D. & Demanet, D. 1993. The importance of surface waves in vibration study and the use of Rayleigh waves for estimating the dynamic characteristics of soils. *Engineering Geology* 34: 105-113.

Lai, C. & Rix, G.J. 1999. Inversion of multi-mode effective dispersion curves. In M. Jamiolkowski, R. Lancellotta, & D. Lo Presti (eds.), *Pre-failure Deformation Characteristics of Geomaterials*. 411-418. Rotterdam: Balkema.

Lai, C.G. 1998. Simultaneous inversion of rayleigh phase velocity and attenuation for near-surface site characterization. Ph.D. dissertation. Georgia Institute of Technology.

Liu, H.-P., Boore, D.M., Joyner, W.B., Oppenheimer, R.E., Warrick, R.E., Zhang, W., Hamilton, J.C. & Brown, L.T. 2000. Comparison of phase velocities from array measurements of Rayleigh waves associated with microtremor and results calculated from borehole shear-wave velocity profiles. *Bulletin of the Seismological Society of America* 90(3): 666-678.

Park, C., Miller, R., & Xia, J. 1999. Multichannel analysis of surface waves. *Geophysics* 64: 800-808.

Rix, G.J., & Lai, C.G. 1998. Simultaneous inversion of surface wave velocity and attenuation. In P.K. Robertson & P.W. Mayne (eds), *Geotechnical Site Characterization*: 503-508. Rotterdam: Balkema.

Rix, G.J., Lai, C.G. & Foti, S. 2001. Simultaneous measurement of surface wave dispersion and attenuation curves. accepted for *Geotechnical Testing Journal.*

Rix, G.J., Lai, C.G. & Spang, A.W. Jr. 2000. In Situ Measurement of Damping Ratio Using Surface Waves. *Journal of Geotechnical and Geoenvironmental Engineering* 126(5): 472-480.

Roësset, J.M., Chang, D.W., & Stokoe, K.H. II. 1991. Comparison of 2-D and 3-D models for analysis of surface wave tests. *5th International Conference on Soil Dynamics and Earthquake Engineering*: 111-126. Karlsruhe, Germany.

Stokoe, K.H., II, Wright, S.G., Bay, J., & Roësset, J. 1994. Characterization of geotechnical sites by SASW method. In R.D. Woods (ed.), *Geophysical Characterization of Sites*: 15-25. New Delhi: Oxford & IBH Publishing Co. Pvt. Ltd.

Tokimatsu, K. 1995. Geotechnical site characterization using surface waves. In K. Ishihara (ed.), *Earthquake Geotechnical Engineering*: 1333-1368. Rotterdam: Balkema.

Tokimatsu, K., Tamura, S., & Kojima, H, 1992. Effects of multiple modes on rayleigh wave dispersion characteristics. *Journal of Geotechnical Engineering* 118(10): 1529-1543.

Zywicki, D.J. 1999. Advanced signal processing methods applied to engineering analysis of seismic surface waves. Ph.D. dissertation. Georgia Institute of Technology.