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2.1 GEOTECHNICAL SOIL CHARACTERIZATION USING FUNDAMENTAL AND HIGHER RAYLEIGH MODES PROPAGATION IN LAYERED MEDIA

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ABSTRACT

The current work focuses on a new method for generating theoretical dispersion curves for Rayleigh wave propagation that is consistent with the common frequency-wavenumber (f-k) technique typically used to determine the experimental dispersion relation. The method considers all possible Rayleigh modes, and will be the basis for an inversion procedure that will lead to the determination of the shear wave velocity profile for any layered system.

Keywords: Rayleigh waves, soil dynamics, geotechnical characterization, dispersion phenomenon

INTRODUCTION

The free vibrations of a stratified medium can be studied by the Stiffness Matrix Method (Kausel and Roesset [1]). Assume *n* horizontally infinite, homogeneous, isotropic, linear elastic layers over a half-space, in addition to harmonic stresses and displacements. If we impose the continuity of displacements and stresses at the interfaces of the layers, and also that the radiation condition holds, the dynamic equilibrium of the whole system can be written in the frequency-wavenumber domain as:

$$\overline{F} = K \cdot \overline{X} \qquad (1)$$

where K is the dynamic stiffness matrix of the global system, and $\overline{X}$ and $\overline{F}$ are the displacements and forces at the interfaces respectively.
If no external loads are applied to the system, the natural vibrations can be found by solving the eigenvalue problem (2.a). Non-trivial solutions can then be found by setting the determinate of K equal to zero as shown in (2.b):

$$0 = K \cdot \overline{X} \qquad \det(K) = C(k, f) = 0 \qquad (2.a, 2.b)$$

The dynamic stiffness matrix K can be obtained by properly assembling the stiffness matrices of the layers and the half-space, the latter opportunely built to account for the radiation condition (Wolf [2]).
Equation (2.b) represents the geometrical Rayleigh dispersion relation and contains all of the pertinent information to describe the system, since it only depends on the geometrical-mechanical properties of the system. For each fixed frequency several wavenumbers may exist, that represent either the generalized Rayleigh modes of the system or both the eigenvalues of the mathematical

problem and the natural modes of vibration. In free vibration conditions only simple waves that satisfy the Rayleigh dispersion relation (2.b) are physically possible.
If by a theoretical point of view several Rayleigh modes can be evaluated, only an apparent averaged dispersion curve can be discerned from the experiments in situ. At this point the problem arises as to which one of the theoretical Rayleigh modes should be chosen to match the experimental dispersion curve, or more generally how to compare the theoretical and the experimental responses of the system in the context of an inversion procedure.
It has been shown by several authors (Stokoe et al. [3], Rix [4], Lai [5], Hebeler [6]) that systems with a regular profile, i.e., stiffness gradually increasing with depth, are normally dispersive and the fundamental or first mode of Rayleigh propagation dominates the system response. In such cases a 2D technique is typically employed in which the higher modes of propagation are neglected over the entire frequency range of interest. However, numerous studies have shown that for irregular or inversely dispersive systems, higher modal behaviour becomes increasing important, and accurate inversions cannot be achieved without including the higher modes in the inversion process. The relative importance of higher modes has also been shown to increase with increasing frequency, (Gucunski and Woods [7], Tokimatsu et al. [8]) and considering only the fundamental mode in such cases often leads to unreliable results. (Rix [4], Hebeler [7]).
There are two common approaches used to account for the contribution of higher Rayleigh modes: 3D techniques and the effective phase velocity method. The 3D techniques consist of numerically simulating the standard SASW test, by numerically evaluating the displacements at the sensors locations on the free surface, i.e., numerically mimicking the experimental procedure. (Joh [9], Ganji et al. [10]) The effective phase velocity method (Lai and Rix [11]) is an elegant way of combining the simplicity of the 2D procedure with the completeness of the 3D procedure. In fact by the definition of the surface of constant phase associated with a composite disturbance travelling on a free surface, an effective phase velocity can be derived that represents an equivalent or apparent phase velocity which contains the information of all Rayleigh modes. As is expected by the dispersion phenomenon of Rayleigh waves the effective phase velocity defines a surface, that is function of the frequency and the distance from the source Some of the advantages offered by the effective phase velocity method are the knowledge of the analytic expressions of the effective phase velocity and its partial derivatives respect to the properties of the system.
In this work a 3D method is proposed (Roma [12]) that guarantees complete consistency with the experimental procedure, and thus mitigating the well-known problem of non-uniqueness in inversion procedures (Lai [5]).

F-K METHOD: EVALUATION OF THE EXPERIMENTAL DISPERSION CURVE

In recent years the f-k method widely used in geophysical field studies has been successfully adopted for near surface soil characterization in the geotechnical scale. (Rix and Lai [11], Zywicki [13], Foti [14], Lai [5], Tselentis and Delis [15], and Mcmechan and Yedlin [16]) The spatial scale of geophysical investigations is large enough for the full dispersion phenomenon of Raleigh modes to take place, and it becomes possible to experimentally measure separated Rayleigh modes. Consequently, the effectiveness of using multiple modes within an inversion procedure is highly increased (Gabriels et al. [17]). However, in the geotechnical scale it is often difficult to distinguish among the Rayleigh modes, since they are still superimposed in a relatively short wave train. The f-k method consists of transforming the wave field measured at N receivers on the free surface from the space time domain to the f-k domain, where it becomes easier to distinguish the several events and isolate individual Rayleigh modes (Doyle [18]). The experimental apparent dispersion curve is then calculated at each frequency from the peak in the spectrum of the vertical displacements by means of: (Gabriels et al. [17], Foti [14], and Zywichi [13],)

$$c_{apparent} = \frac{\omega}{k_{\max}} \tag{3}$$

where k_{max} is the wavenumber corresponding to the maximum in the spectrum at the frequency ω. Depending on whether the source is harmonic or impulsive two different procedures can be employed to evaluate the experimental dispersion curve. If the source is harmonic, each frequency in the system is excited at a point on the free surface for a time long enough to induce a steady-state response. The measurements are recorded at N receivers located on the free surface in a linear array. The distance D between the source and the first sensor, the spacing interval Δx between 2 adjacent sensors, and the total length L covered by the array are important factors in determining the Nyquest wavenumber and the wavenumber resolution

$$k_{Nyquist} = \frac{2\pi}{2\Delta x} \qquad \Delta k = \frac{2\pi}{L} \qquad L = (N-1)\cdot \Delta x + D \qquad \text{(4.a),(4.b),(4.c)}$$

where L is the total distance between the source and the last receiver.
Similar considerations concerning frequency resolution and the Nyquest frequency must be considered in the frequency domain. Usually a temporal sampling rate of Δt = 2ms is chosen, resulting in a Nyquest frequency of, $f_{Nyquest}$=250Hz, which is much higher than the tyipical frequency range of interest in geotechnical investigations (0-100 Hz). By means of the measurements at all sensors the power spectrum of the displacements is evaluated and the peak in the power spectrum is determined to evaluate the apparent phase velocity using equation (3). The details of the procedure can be found in Zywichi [13] and Hebeler [6]. If the source is impulsive a 2D Fourier Transformation of the field displacements (5) from time-space to frequency-wavenumber furnishes the spectrum of displacements (6), where it becomes straightforward to identify the Rayleigh apparent dispersion curve as the peaks in the spectrum (McMechan and Yedlin [16]).

$$u(x,t) = \iint \frac{N(k,\omega)}{R(k,\omega)} \cdot e^{i(kx-\omega t)} \cdot d\omega \cdot dk \tag{5}$$

$$\overline{\overline{u}}(k,\omega) = \iint u(x,t) \cdot e^{i\omega t} \cdot e^{-ikx} d\omega \cdot dk = \frac{N(k,\omega)}{C(k,\omega)} \tag{6}$$

where $N(k,\omega)$ is a function of the exciting source and $C(k,\omega)$ is the Rayleigh dispersion function found in (2.b).
It should be observed that under the assumptions of a linear time invariant system and a point source the use of either an impulsive or a harmonic source gives the same apparent dispersion curve. This result should be expected if the dispersion curve is considered an intrinsic property of the system under investigation. The hypothesis that different sources produce the same apparent dispersion curve has been both numerically demonstrated and experimentally confirmed (Roma [19]). It has also been found that harmonic sources provide better signal to noise ratios and allow for higher energy generation at low frequencies. (Hebeler [6], Rix [4]) The f-k method has revealed to be a powerful tool for the calculation of experimental dispersion curves, overcoming all the drawbacks presented by the standard SASW method, as shown in: Foti [14], Hebeler [6], and Zywicki[13].

F-K METHOD: EVALUATION OF THE THEORETICAL DISPERSION CURVE

Once the experimental apparent dispersion curve has been calculated a proper theoretical dispersion curve is needed for comparison in the framework of an inversion procedure. A theoretical method consistent with the recent experimental multi-station f-k method is now proposed (Roma [12]). Consider the displacements in the space-time domain due to a harmonic point source on the free surface given by (Aki and Richards [20]):

$$\mathrm{u}(x,z,\omega,t)=\sum_{j=1}^{M}\left[A(x,z,\omega)\right]_j\cdot e^{i\left(\omega\cdot t-k_j\cdot x\right)} \tag{7}$$

If we eliminate the exponential term $e^{i\omega t}$, the Transfer Function (or Green's Function) of the system in the frequency-space domain is obtained (8.a), and if we also apply a 1D Fourier Transformation from frequency-space to frequency-wavenumber, then the spectrum of displacements can be calculated (8.b):

$$\bar{\mathrm{u}}(x,z,\omega)=\sum_{j=1}^{M}\left[A(x,z,\omega)\right]_j\cdot e^{-ik_j\cdot x} \tag{8.a}$$

$$\bar{\bar{\mathrm{u}}}(k,\omega)=\int\sum_{j=1}^{M}A_j(x,\omega)\cdot e^{i(k-k_j)\cdot x}\cdot dx \tag{8.b}$$

The remainder of the theoretical procedure follows the procedure for calculating the experimental dispersion curve using the aforementioned f-k method. That is the maximum in the spectrum is determined for a series of fixed frequencies and the consequent apparent phase velocity is evaluated according to (3). The advantages of the proposed method are:

a) There is no need for characterization of the source.
b) Only a 1D Fourier Transformation is performed, removing the necessity to transform to and from the space-time domain.
c) Perfect consistency exists between the experimental and the theoretical procedures.

In fact, it is possible to use the same array configuration and calculation procedure in both the experimental and theoretical calculation procedures. Point (a) is proven above in the discussion regarding the non-uniqueness of the source in the calculation of the apparent dispersion curve (Roma [19]). Point (b) allows for greatly increased numerical efficiency within the proposed inversion algorithm. Point (c) is significant because consistency between the experimental and the theoretical procedures mitigates some of the ambiguity of the inversion process. An important feature to be outlined is that the dispersion curve observed in the experiments and hence in the theoretical simulation is not only the response of the system, but is also influenced by the configuration of sensors. (Roma [12], Hebeler [6], and Zywicki[13]) In fact the peaks in the spectrum are the result of an interaction between the system and the array response function, especially if relative maxima are considered at each frequency. (Roma [12]) Consequently, it becomes a priority to adopt the same configuration in the theoretical calculations when simulating the in-situ experiment.

COMPARISON AMONG DIFFERENT PROCEDURES: VALIDATION OF THE METHOD

In order to assess the validity of the new proposed method for evaluating the theoretical dispersion curve, some artificial systems have been considered that are representative of the different types of sites typically encountered in the geotechnical scale. Tables from 1 to 4 illustrate the characteristics of sites A-D respectively.

Layer	**h(m)**	**V_P (m/s)**	**V_S (m/s)**	**ρ (Kg/m^3)**
1	5	600	350	1800
2	10	700	400	1800
Half-space	∞	800	450	1800

Table 1: System A, normally dispersive site.

Layer	**h(m)**	**V_P (m/s)**	**V_S (m/s)**	**ρ (Kg/m^3)**
1	5	700	400	1800
2	3	500	300	1800
Half-space	∞	800	450	1800

Table 2: System B, inversely dispersive: softer layer between two stiffer ones.

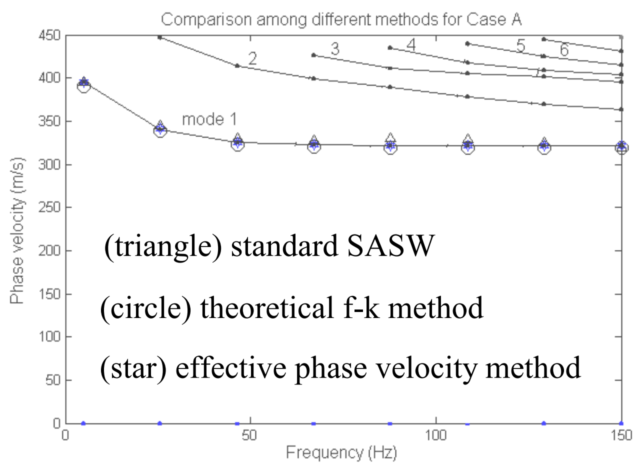
Layer	**h(m)**	**V_P (m/s)**	**V_S (m/s)**	**ρ (Kg/m^3)**
1	10	1500	1000	1900
2	10	750	500	1900
Half-space	∞	1500	1000	1900

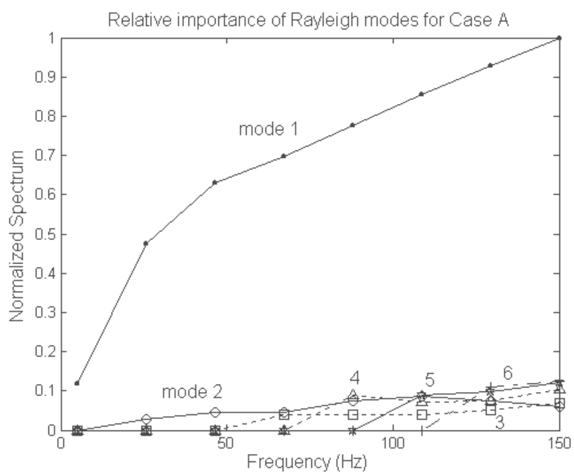
Table 3: System C, inversely dispersive: stiff surface layer

Layer	**h(m)**	**V_P (m/s)**	**V_S (m/s)**	**ρ (Kg/m^3)**
1	20	1200	600	1800
2	20	2000	1000	1800
3	80	1200	600	1800
Half-space	∞	2000	1000	1800

Table 4: System D, From Gucunski and Woods [22], inversely dispersive: stiff layer between two softer layers.

Figures 1a, b: Comparison among different methods of evaluating the theoretical apparent dispersion curve (a) and relative importance of Rayleigh modes (b) for case A.





In case A, the stiffness gradually increases with depth and the phase velocity is greater than the group velocity at all the frequencies, so the system is said to be normally dispersive. In this circumstance the fundamental Rayleigh mode dominates the dispersion curve that would be measured in field. In fact all the apparent curves simulated following standard SASW, effective

phase velocity and f-k methods coincide with the fundamental mode (Figure 1a). Hence, as expected when dealing with such systems, using only the fundamental mode provides reliable results and more complicated analyses involving multi-modal analysis are not warranted. Figure 1.b shows the normalized spectrum of the vertical displacements for each mode independently.
The curves shown in Figure 1.b represent the projections on the frequency-amplitude plane of the spectrum associated with each mode. In practice each modal spectrum is traced in the frequency-wave number domain along its modal path. This representation allows for the relative importance of all Rayleigh modes to be more easily understood. In fact from Figure 1b it can be observed that the fundamental mode is always predominant, thus explaining the behavior of the apparent theoretical dispersion curve.

Figures 2a, b: Theoretical apparent dispersion curve (a) and relative importance of Rayleigh modes (b) for case B.

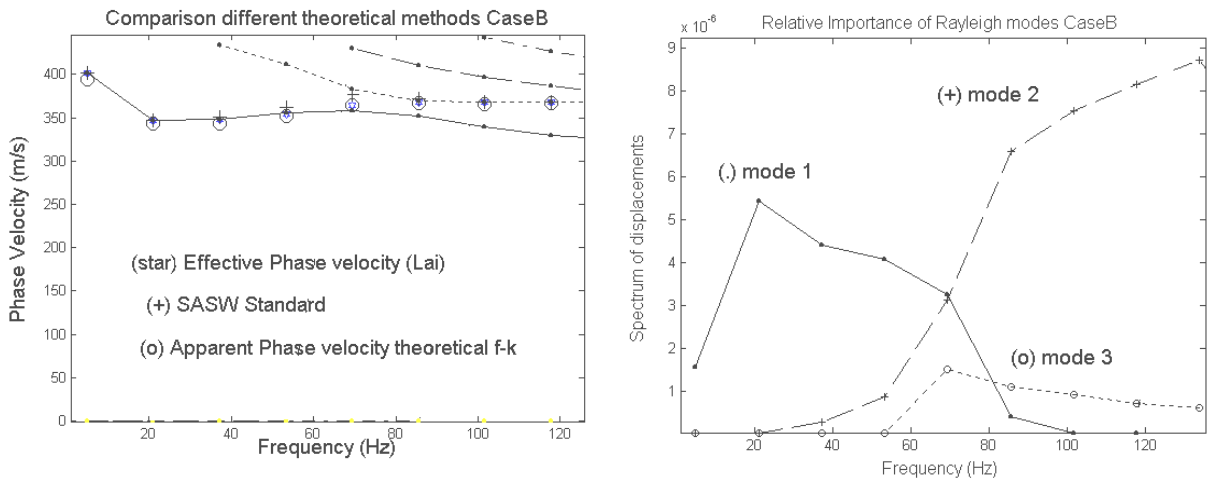
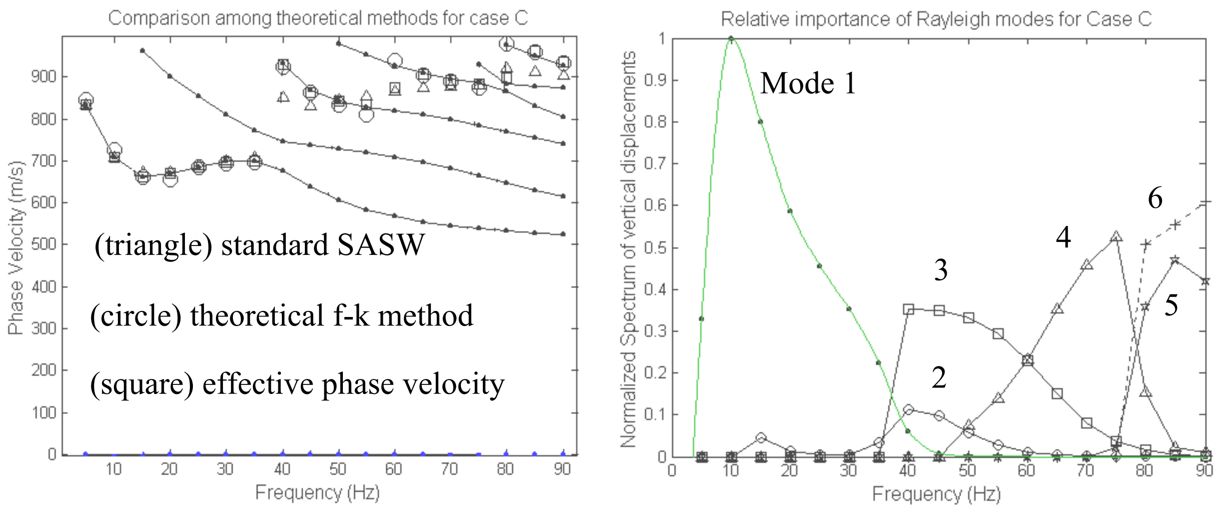


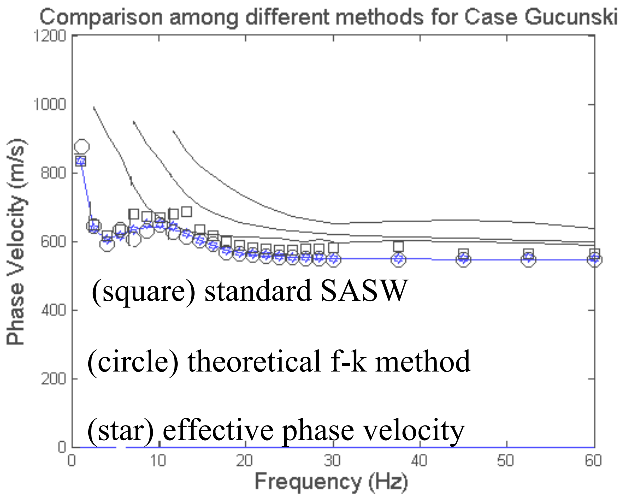
Fig.3a, b: Theoretical apparent dispersion curve (a) and relative importance of Rayleigh modes (b) for case B.

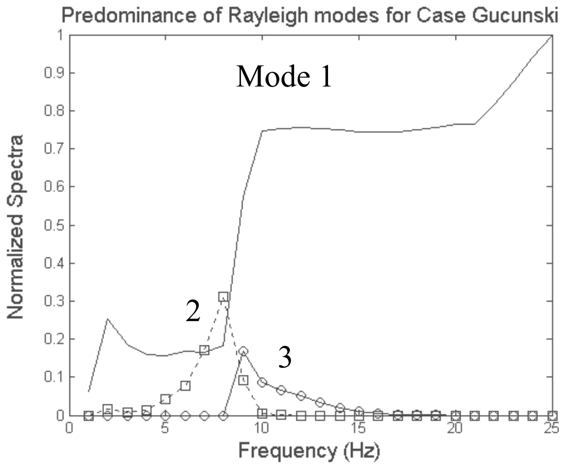


Case B represents a profile with a soft layer between two stiffer layers. Since it is an inversely dispersive system higher Rayleigh modes should not be neglected. In fact at a frequency of 70Hz the fundamental mode looses its predominance and the simulated apparent dispersion curve follows the 2nd Rayleigh mode (Figure 2a). This can again be easily explained by looking at the normalized

spectra in Figure 2b. Even in Case B, the theoretical apparent dispersion curve obtained with the proposed method closely matches the dispersion curves obtained with other methods.
System C has a profile characterized by a stiffer layer on the free surface underlian by softer layers, as such it is also an inversely dispersive system as can be deduced from the simulated dispersion curves shown in Figure 3a. In this Case, the transition of predominance from the fundamental mode to the higher modes is more evident than in case B, in fact the first four modes are involved in the frequency range shown.
It is interesting to observe that from the normalized spectra of each independent mode, the natural frequencies and wave numbers of the system can be individually discerned (Figure 3b). Each mode reaches at least one distinct peak along its path in the frequency-wavenumber domain, where the response of the system on the free surface is maximized. The position of the peaks in the spectrum identifies the frequencies and wavenumbers of resonance for the system. Additionally these peaks only depend on the geometrical-mechanical properties of the system, and not on the sensor array. (Roma [12]) A sensitivity analysis has been recently conducted, where only one layer over a half-space was considered. (Roma et al. [21])

Fig.4a, b: Theoretical apparent dispersion curves (a) and normalized spectra modal vertical displacements (b) for case from Gucunski and Woods [22].





The last Case chosen is an example from Gucunski and Woods [22] that has a profile with a stiffer layer located between two softer layers. Although it is not evident, the system is not normally dispersive since the 2nd mode becomes dominant within a range of frequencies eqaul to (5Hz-10Hz) (Figure 4a and Figure 4b). As in the previous Cases the agreement among the standard SASW, the effective phase velocity and the new proposed method is very satisfactory. From the above examples three conclusions can be inferred:

1) A 2D analysis using only the fundamental mode is acceptable when dealing with normally dispersive systems.
2) Higher modes cannot be ignored when inversely dispersive systems are of concern.
3) The new method proposed to theoretically evaluate the apparent dispersion curve has shown to be valid in all types of systems, giving results in excellent agreement with other methods.

APPLICATION TO A TEST SITE

Once the validity of the new method was verified it was integrated into an inversion procedure to simulate the forward propagation of Rayleigh waves and generate the theoretical apparent dispersion curve. Herein the results of the inversion process applied to a test site will be presented, and the reader is referred to other works for more details about the inversion algorithm. (Roma [12; 23]) The algorithm is generally described as a non-linear constrained optimization algorithm, whose type is of that proposed by Davidon–Fletcher-Powell [24], and which belongs to the class of Quasi-Newton.

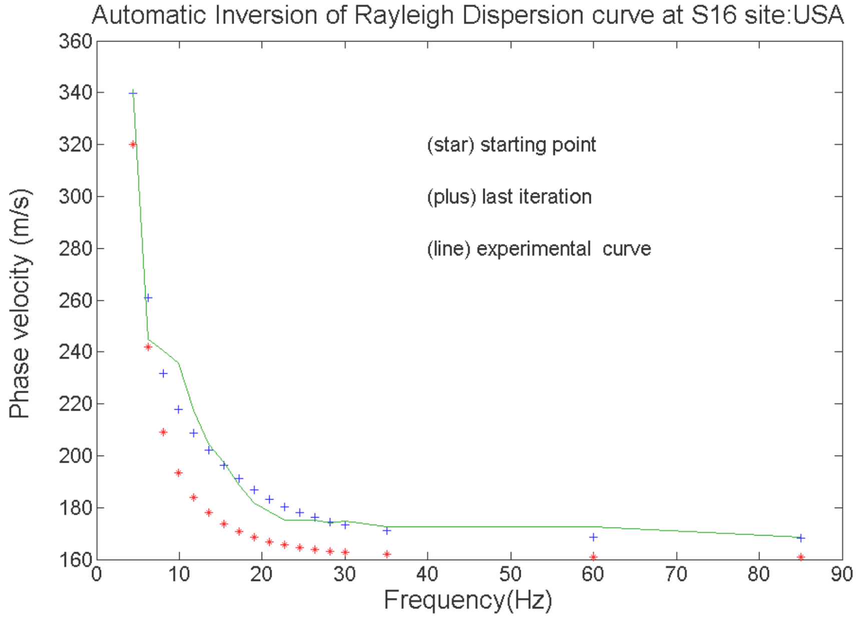
The teest presented is designated as Street 16 and it is located near Memphis, TN USA. The soil in the area is of Holocene age as the site lies within the flood plain of Nonconnah Creek, with the water table ranging in depth from 2m to 4m A harmonic electromagnetic source and 16 ultra low frequency Wilcoxon 731A seismic accelerometers were used in an array configuration optimized for spatial sampling. (Hebeler [6]) .

Table 5: Inversion of the shear wave velocity profile at Street 16 site.

Layer	h(m)	Poisson's ratio	ρ (Kg/m^3)	V_S (m/s) Starting profile	V_S (m/s) Inverted profile
1	3.5	0.15	1900	180	188
2	3	0.45	1900	200	244
3	5	0.45	1900	250	279
4	3.5	0.45	1900	300	279
5	5.5	0.45	1900	400	377
6	9.5	0.45	1900	500	482
7	7.7	0.45	1900	600	597
Half-space	-	0.45	1900	800	796

The apparent dispersion curve has been obtained by selecting the peaks in the power spectrum and has been plotted in figure 5 (green line). The strategy that has been adopted for the inversion procedure consists of fixing the thickness of the layers and inverting for the shear wave velocities. This procedure does not cause any loss of validity, since two or more layers may be found with the same velocity as is the case for layers 3 and 4 in Table 5. The characteristics of the site are also shown in Table 5. The Poisson's ratios take into consideration the existence of the water table at a depth of about 3.5m, and the mass densities ρ have been fixed. The starting shear wave velocity, as well as the inverted one are also reported. In figure 5, the apparent dispersion curves of the initial V_S profile (red star) and the inverted configuration (blue plus) have been plotted. As observed, the theoretical dispersion curve of the last (11^{th}) iteration closely mathces the experimental dispersion curve.

Figure 5: Experimental (green-line) and theoretically simulated dispersion curves for Street 16 site.



CONCLUSIONS

A new procedure has been proposed to theoretically simulate the experimental f-k method in evaluating the apparent dispersion curve for Rayleigh waves in a stratified medium. The method allows for all higher Rayleigh modes to be accounted for and its validity has been proven for both normally and inversely dispersive systems by means of a comparison with standard SASW and effective phase velocity methods. Some aspects about the relative importance of Rayleigh modes have been illustrated, and the frequencies and the wavenumbers of resonance of a layered system have been recognized. The consistency between the experimental and the theoretical procedures improves the solution to the non-unique problem of inverting real profiles. A test site has been successfully inverted by means of an optimization algorithm based on the presented theoretical simulation of Rayleigh wave propagation.

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