Automated Inversion of Rayleigh Geometrical Dispersion Relation for Geotechnical Soil Identification

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ABSTRACT

Soil identification means determining the stratigraphy, the water table position, the stiffness and the damping profiles. In this work the Multichannel SASW method is proposed for evaluating the shear wave velocity profile and the stratigraphy. The identification problem is mathematically represented by a non-linear constrained optimization problem. In fact the system parameters are searched to minimize the distance between the experimental and the theoretical responses of the layered medium. The Davidon-Fletcher-Powell (DFP) algorithm is used for this task. The effectiveness of the procedure is shown with reference to a real case.

1. INTRODUCTION

In the treatment the following hypotheses will be assumed: (1) the system consists of a set of horizontally infinite layers overlaying an infinite half-space, (2) the medium is considered as an equivalent continuum, in which the layers are supposed linear elastic, homogeneous, monophase, isotropic, (3) each layer is characterized by the thickness h, the mass density ρ, the shear wave velocity V_S and the Poisson ratio ν. The mass density ρ will be considered as a known parameter and the Poisson ratio ν reflects the existence and the position of the water table in the layered medium.

2.1 Description of the Multichannel SASW Method

The multichannel SASW method (Rix G. et al., 2001) is a non-invasive technique, that consists of perturbing the medium at a point on the free surface and measuring the travelling perturbation at several stations on the free surface. The main contribution to the surface motion is given by the Rayleigh waves, whose speed depends on the stiffness of the sampled portion of the system. In a layered medium Rayleigh waves are subjected to geometrical dispersion, i.e. waves of different wavelength travel at different phase and group velocities (Achenbach, J.D., 1999, Aki, K. and Richards, P.G., 1980) . As a consequence the apparent phase velocity of the propagating disturbance depends on the frequency, as it is actually measured in the field or as it is theoretically predicted (see fig. 1). The higher the frequency of excitation, the shorter the wavelength and the shorter the depth sampled by the Rayleigh waves. Conversely, the lower the frequency, the longer the wavelength and the deeper the layers investigated.

The Identification procedure is made of three steps: (1) the first step is the experiment in situ. By measuring the particle velocities or the particle accelerations the experimental system response is calculated in terms of *Geometrical Dispersion Relation* of Rayleigh waves, (2) the second step is the numerical simulation of the experimental test, so that a consistent theoretical dispersion relation is evaluated, (3) the third step consists of iteratively varying the mechanical and geometrical properties of the system, until an optimal match is obtained between the experimental and the theoretical system responses. This task can be accomplished by either a trial-and-error approach or an automated optimization procedure.
We will focus on the optimization problem and the reader is referred to other works for the first two steps (Roma V., 2001, Roma V. et al., 2002).

2.2. Optimization Algorithm

Once both the experimental and the theoretical Dispersion Relations of Rayleigh waves have been determined, the nonlinear constrained optimization problem can be stated as follows:

$$\text{minimum of } e = \sum_{j=1}^{N} \left[V\exp_j - Vtheo_j(h_i, Vs_i) \right]^2 \qquad (1)$$

subjected to the constraints: $h_i \geq h_{min}$ i = 1, 2,..., M (2)

$$\left(\sum_{i=1}^{M} h_i \right) \leq z_{max} \quad (3), \qquad Vs_i > 0 \quad i = 1, 2, ..., (M+1) \quad (4)$$

in which N is the number of frequencies, M is the number of layers, $V_{exp}(j)$ and $V_{theo}(j)$ are respectively the experimental and theoretical apparent phase velocities at the j^{th} frequency, *e* is the least square error of the experimental and theoretical curve fitting, h_i is the thickness of the i^{th} layer. For a problem with M layers , (M+1) shear wave velocities exist, namely M shear velocities for the M layers plus one shear wave velocity for the half-space.
The error in (1) is a strongly non-linear function of the geometrical and mechanical properties of the layered medium and represents the distance between the experimental and the theoretical apparent dispersion curves. The linear conditions (2)÷(4) indicate the constraints of the physical problem and they delineate a convex domain of feasibility for the parameters h_i and Vs_i. The condition (2) imposes a minimum thickness at each layer, since a thinner layer would not have a significant influence on the global experimental response. The condition (3) states that a maximum depth exists, down to which information can be obtained by the method. The condition (4) says that negative shear wave velocities do not have any physical sense. The non-linear constrained optimization problem (1)÷(4) is solved by means of a penalty method (Reklaitis et al., 1996). The mathematical problem is transformed into an infinite series of unconstrained optimization problems:

$$F(h_i, vs_j, R) = \sum_{j=1}^{N} \left[V\exp_j - Vtheo_j(h_i, vs_j) \right]^2 +$$

$$+ R \cdot \left[\left\{ \sum_{i=1}^{M} \left[\min(h_i - h_{min}, 0) \right]^2 \right\} + \left[\min\left(- \sum_{i=1}^{M} h_i + z_{max}, 0 \right) \right]^2 + \left\{ \sum_{i=1}^{M+1} \left[\min(Vs_j, 0) \right]^2 \right\} \right] \qquad (5)$$

where $\min(k,0) = \begin{cases} k & k \leq 0 \\ 0 & k \geq 0 \end{cases}$ (6)

The R parameter is a penalty factor that combines the " distance " between the experimental and the theoretical responses with the constraints.
The terms multiplied by the factor R inside the square bracket represent the penalties associated to the constraints of the initial optimization problem.
The series of unconstrained, non-linear, optimization problems converges to the correct solution for $R \rightarrow \infty$. For each iteration with a fixed R the minimization of F is an unconstrained non-linear optimization problem. By varying the R parameter a different importance is given to the "distance" respect to the constraints. The objective function $F(h_i, vs_i, R)$ is minimized by using the Davidon–Fletcher-Powell (DFP) method (Reklaitis et al.,1996). This algorithm is a Quasi-Newton algorithm and as such it can be classified as a local-search technique. The optimization procedure has been implemented by the Author into a software *Tremor* and tested on both numerical examples and real sites (Roma V., 2001). The main results are summarized below:

1) The contemporary identification of the thickness and the shear wave velocity profile may lead to unreliable results, because of the existence of local minima of the objective function in the domain of the feasible parameters.
2) The identification of the shear wave velocities is successful if the thickness are kept as constant during the optimization procedure.
3) The inversion of the stratigraphy gives excellent results if the shear wave velocities are fixed at constant values.
4) The objective function is more sensitive to the variation of the shear wave velocities of the top layer and of the half-space.

3. Application to a real site: Houston Levee Park, Memphis, Tennessee, USA

As a real case the site Houston Levee Park in Germantown, Memphis, Tennessee, USA has been investigated with the proposed method. This site is located nearby the Wolf river and unfortunately there is no available information from alternative methods.
The experimental test has been performed by using a vertical harmonic shaker, operating in the range of frequency of interest, that is between 5Hz and 100 Hz. In order to evaluate the experimental dispersion curve (see blue plusses in fig. 1) the vertical component has been considered and the procedure explained in (Roma et al., 2002) has been followed. The experimental phase velocity is quite smooth and its behavior resembles the typical tendency of normally dispersive sites in which the stiffness monotonically increases with depth.
In the Identification process (Inversion of the parameters) the thicknesses have been chosen as fixed parameters as well as the mass density ρ=1800kg/m^3 . The water table position is a variable which influences the value of the Poisson ratio ν (see fig. 3). Above the water table ν=0.2, instead below the water table ν=0.48, because of the almost incompressibility of the saturated medium. Also the shear wave velocities of each layer and the half-space have been adopted as unknowns. Two different shear wave velocity profiles have been assumed as possible starting configurations for the Inversion process. They are reported in table 1 as well as the correspondent inverted shear wave velocities at the end of the Identification procedure. The finally inverted shear wave velocity profiles are quite similar and from a geotechnical engineering viewpoint they represent the same optimal solution within a small range of uncertainty (see fig. 3).

				Case	A	Case	B
Layer	**h (m)**	**ρ (kg/m^3)**	**ν**	**Initial Vs (m/s)**	**Final Vs (m/s)**	**Initial Vs (m/s)**	**Final Vs (m/s)**
1	2	1900	0.2	140	152	160	153
2	2	1900	0.48	200	226	230	214
3	2	1900	0.48	210	233	280	252
4	2	1900	0.48	280	288	305	284
5	3	1900	0.48	300	300	325	289
6	3	1900	0.48	325	320	335	311
7	3	1900	0.48	330	325	340	329
Half-space	∞	1900	0.48	365	353	345	357

Table 1. Results of the Identification procedure for the real site Houston Levee Park in Germantown, Memphis, Tennessee, USA. The same shear wave velocity profile is obtained, starting from two different initial configurations Case A and Case B.

From the values of the *Objective Function* and of the gradient of the Objective Function it can be realized that the final configuration founded by the algorithm is the optimal solution. After just 5 iterations the *Objective Function* in both the Cases A and B has lower down significantly (see fig. 2).

	Case	**A**	**Case**	**B**
	Initial	**Final**	**Initial**	**Final**
Objective Function	4800	138	3115	102
Absolute value of the Gradient of the Objective Function	182219	1.6	81807	30.8
Number of iterations	25		17	

Table 2. Performance of the Identification Procedure.

4. CONCLUSIONS

Soil Identification can be automatically accomplished by means of the software *Tremor* made by the Author. The software is based on the Multichannel SASW procedure and the DFP Quasi-Newton optimization algorithm. The method, though some limitations, presents many advantages for engineering applications, especially when dealing with soil-structure dynamic interactions (Clough R.W. and Penzien J., 1993). As an example the ground profile at Houston Levee Park in Memphis (Tennessee, USA) has been successfully identified.

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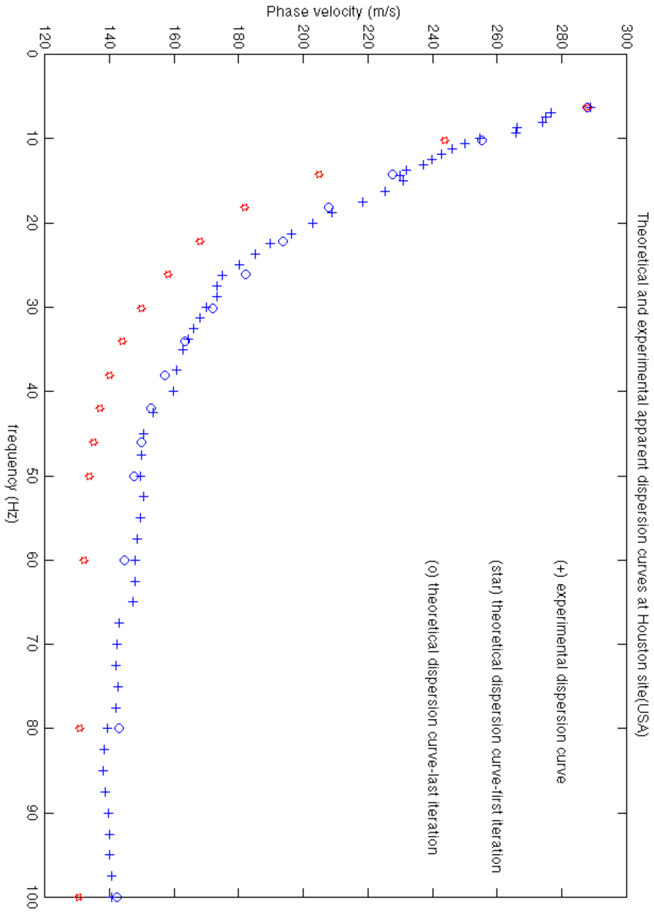

Figure 1. *Geometrical Dispersion Relation* of Rayleigh waves in Houston Leeve Park (Memphis, USA). Experimental curve (blue croxes), theoretical curve for the starting configuration (red stars) and theoretical curve (blue circles, Case A) at the end of the Inversion process.

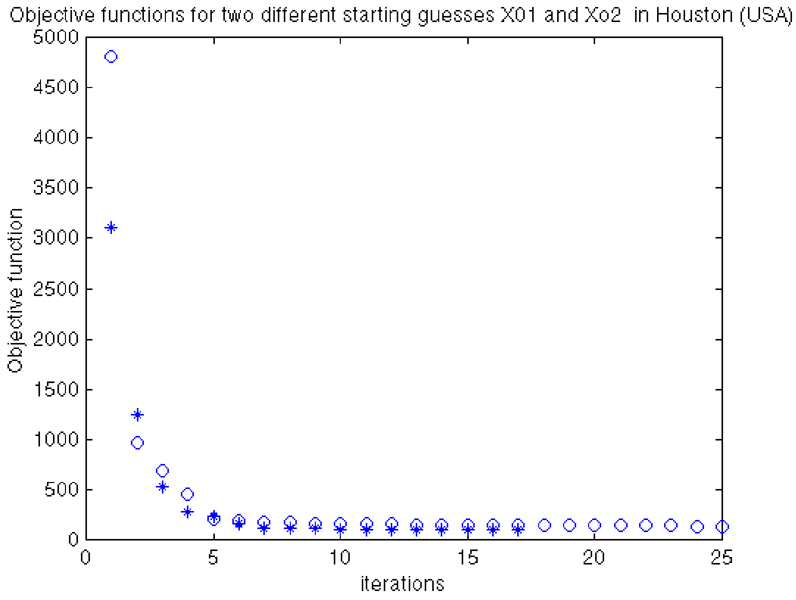

Figure 2. Behavior of the Objective Function in the Cases A (circles) and B (stars) for the Identification of the real site Houston Levee Park (Memphis, Tennessee, USA).

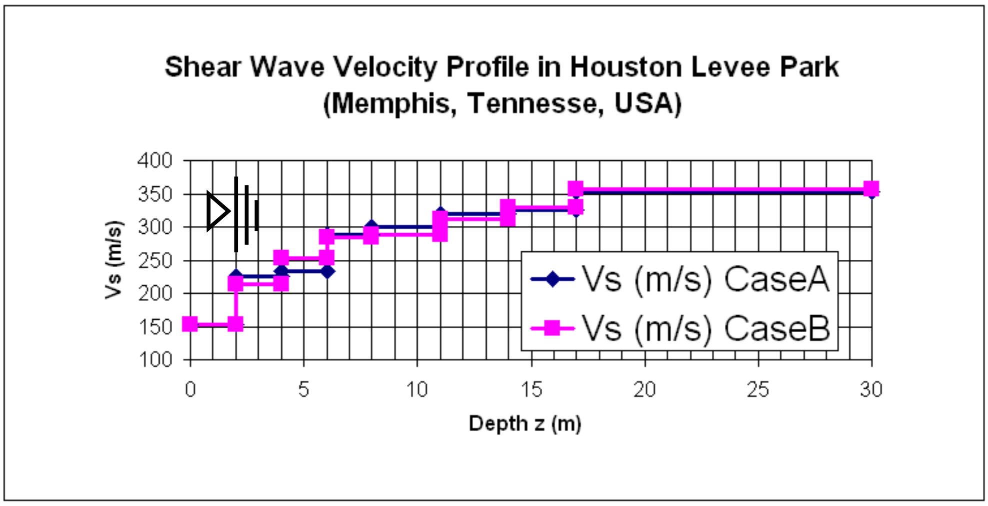

Figure 3. Soil Identification at Houston Levee Park (Memphis, Tennessee, USA). Comparison between the two independent results.